UniBwM – EIT Prof. Dr.-Ing. habil. H. Dalichau	Formelsammlung Mathematik

Diese Formelsammlung dient als Ergänzung/Erweiterung einer mathematischen Standard-Formelsammlung wie z.B. Bronstein-Semendjajew oder Abramowitz-Stegun unter besonderer Berücksichtigung der Belange der HF- und μλ-Technik.

I. Näherungen

$$x \ll 1$$

$(1 \pm x)^n \approx 1 \pm nx$

$\sqrt{1 \pm x} \approx 1 \pm x/2$

$\dfrac{1}{1+x} \approx 1 - x$

$1\big/\sqrt{1+x} \approx 1 - x/2$

$(1+x_1)(1+x_2) \approx 1 + x_1 + x_2$

$\dfrac{1+x_1}{1+x_2} \approx 1 + x_1 - x_2$

$e^x \approx 1 + x$

$a^x \approx 1 + x \ln a$

$\ln(1+x) \approx x$

$\ln\dfrac{1+x}{1-x} \approx 2x$

$\ln\left(x + \sqrt{x^2+1}\right) \approx x$

$\cos x \approx 1$

$\sin x \approx x$

$\cosh x \approx 1$

$\sinh x \approx x$

$\arcsin x \approx x$

$\arctan x \approx x$

$|\cosh(x + jx)| \approx 1$

$|\sinh(x + jx)| \approx 0$

$\arccos\dfrac{1}{1+x} \approx \sqrt{2x}$

$$x \gg 1$$

$\ln x \approx ar\cosh\left(x/2\right)$

$$x > 2$$

$|\cosh(x + jx)| \approx e^x/2$

$|\sinh(x + jx)| \approx e^x/2$

II. Hyperbolische Funktionen

$$\sinh^2 x + \frac{1}{2} = \cosh^2 x - \frac{1}{2}$$

$$\sinh^2 x + \frac{1}{2} = \frac{1}{2}\cosh(2x)$$

$$\sinh^2 x + \frac{1}{2} = \frac{1}{2}\frac{1+\tanh^2 x}{1-\tanh^2 x}$$

$$\cosh^2 x - \frac{1}{2} = \frac{1}{2}\cosh(2x)$$

$$\frac{1+\tanh^2 x}{1-\tanh^2 x} = \cosh(2x)$$

$$\frac{1}{2}\left(\tanh x + \frac{1}{\tanh x}\right) = \frac{1}{\tanh(2x)}$$

$$\frac{\tanh x - 1}{\tanh x + 1} = -e^{-2x}$$

$$\frac{1}{\tanh x} - \tanh x = \frac{2}{\sinh(2x)}$$

$$\cosh x + \sinh x = e^x$$

$$\cosh^2 x + \sinh^2 x = \cosh(2x)$$

$$\sinh(x)\ln(y) = \sinh\left\{\ln\left(y^x\right)\right\} = \frac{y^x - y^{-x}}{2}$$

$$ar\cosh x = \ln\left(x + \sqrt{x^2-1}\right)$$

$$2 \operatorname{arcosh} x = \operatorname{arcosh}(2x^2-1)$$

$$ar\cosh x = ar\sinh\sqrt{x^2-1}$$

$$ar\cosh\left\{\frac{1}{2}\left(x+\frac{1}{x}\right)\right\} = \ln x$$

$$ar\cosh\sqrt{x} = \ln\left(\sqrt{x} + \sqrt{x-1}\right)$$

$$ar\cosh x = j\arccos x \qquad \textit{für } x < 1, \textit{ reell}$$

$$\arcsin x = \frac{\pi}{2} + j\,ar\cosh x \qquad \textit{für } x > 1, \textit{ reell}$$

$$ar\cosh\sqrt{x} = j\arccos\sqrt{x} \qquad \textit{für } x < 1, \textit{ reell}$$

$$ar\tanh x = \mathrm{ar}\tanh\frac{1}{x} + j\pi \qquad \textit{für } x > 1, \textit{ reell}$$

$$\arctan\left(\frac{1}{\tan x}\right) = \frac{\pi}{2} - x$$

$$ar\tanh\frac{1-b}{1+b} = -\frac{1}{2}\ln b$$

$$\tanh(\ln x) = \frac{x^2-1}{x^2+1}$$

$$\ln x = ar\sinh\left\{\frac{1}{2}\left(x-\frac{1}{x}\right)\right\}$$

$$x - \frac{1}{x} = 2\sinh(\ln x)$$

$$\ln x = ar\cosh\left\{\frac{1}{2}\left(x+\frac{1}{x}\right)\right\}$$

$$x+\frac{1}{x} = 2\cosh(\ln x)$$

$$\frac{\exp(w\pi)-1}{\exp(w\pi)+1} = \tanh\left(\frac{w\pi}{2}\right)$$

$$\frac{1+\exp(w\pi)}{1-\exp(w\pi)} = -1\Big/\tanh\left(\frac{w\pi}{2}\right)$$

III. Trigonometrische Funktionen

$$\frac{1-e^{j\varphi}}{1+e^{j\varphi}} = j\tan\frac{\varphi}{2} \qquad \ln\frac{e^{j\varphi}+1}{e^{j\varphi}-1} = \frac{1}{2}\ln\frac{1+\cos\varphi}{1-\cos\varphi} + j\frac{\pi}{2}$$

$$\frac{1+jx}{1-jx} = e^{j2\arctan x} \qquad \sqrt{\frac{jx-1}{jx+1}} = e^{j\arctan x}$$

$$\frac{1-\tan^2 x}{1+\tan^2 x} = \cos(2x) \qquad \frac{1}{2}\left(\tan x + \frac{1}{\tan x}\right) = \frac{1}{\sin(2x)}$$

$$\frac{1}{\tan x} - \tan x = \frac{2}{\tan(2x)} \qquad a\cos\varphi + b\sin\varphi = \sqrt{a^2+b^2}\cos\left(\varphi - \arctan\frac{b}{a}\right)$$

$$\tan x + \frac{1}{\cos x} = \frac{1+\sin x}{\cos x} = \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}$$

$$\sin\left(n\arccos\left[ze^{j\pi}\right]\right) = -j\left(-1\right)^n\frac{x^n-x^{-n}}{2} \quad \textit{mit} \quad x = z+\sqrt{z^2-1} \quad \textit{und} \quad z>1$$

$$\arctan\left(e^{j\varphi}\right) = \frac{\pi}{4} - j\frac{1}{4}\ln\frac{1-\sin\varphi}{1+\sin\varphi}$$

$$\frac{1}{2}\arctan x = \arctan\frac{\sqrt{x^2+1}-1}{x} \qquad \frac{1}{2}\operatorname{arc\,cot} x = \arctan\left(\sqrt{x^2+1}-x\right)$$

$$x = \sin^2\varphi + x^2\cos^2\varphi \qquad \rightarrow \qquad x = \tan^2\varphi$$

IV. Umformungen

$$y = \frac{1-x}{1+x} \quad \rightarrow \quad x = \frac{1-y}{1+y}$$

$$y = \frac{a+x}{a-x} \quad \rightarrow \quad x = a\,\frac{y-1}{y+1}$$

$$\frac{x-y}{x+y} = -\frac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x}+\frac{1}{y}}$$

$$x(x+2) = (1+x)^2-1$$

$$\frac{\sqrt{D^2-d^2}+(D-d)}{\sqrt{D^2-d^2}-(D-d)} = \frac{D}{d}+\sqrt{\left(\frac{D}{d}\right)^2-1}$$

$$\frac{1+e^x}{1+e^{-x}} = e^x$$

$$\frac{1}{x-a}-\frac{1}{x+a} = \frac{2a}{x^2-a^2}$$

$$\ln\left(\sqrt{2}+1\right) = -\ln\left(\sqrt{2}-1\right)$$

$$x+\frac{1}{x} = 2y \quad \rightarrow \quad x_{1,2} = y \pm \sqrt{y^2-1}$$

$$x+\sqrt{x^2-1} = \frac{1}{x-\sqrt{x^2-1}}$$

$$x-\sqrt{x^2-1} = \frac{1}{x+\sqrt{x^2-1}}$$

$$\left(x-\frac{1}{x}\right)\left(y-\frac{1}{y}\right) = \left(xy+\frac{1}{xy}\right)-\left(\frac{x}{y}+\frac{y}{x}\right)$$

$$\left(x+\frac{1}{x}\right)\left(y-\frac{1}{y}\right) = \left(xy-\frac{1}{xy}\right)+\left(\frac{y}{x}-\frac{x}{y}\right)$$

$$\ln\left(\frac{a+b}{a-b}\right) = 2\ln(a+b)-\ln\left(a^2-b^2\right)$$

V. Komplexe Zahlen

$$|\cosh(x+jx)| = \sqrt{\frac{1}{2}\cosh(2x) + \cos^2 x - \frac{1}{2}}$$

$$|\sinh(x+jx)| = \sqrt{\frac{1}{2}\cosh(2x) + \sin^2 x - \frac{1}{2}}$$

$$\tanh(\mathrm{u} + \mathrm{j\,v}) = \frac{\sinh(2\mathrm{u}) + \mathrm{j}\sin(2\mathrm{v})}{\cosh(2\mathrm{u}) + \cos(2\mathrm{v})}$$

$$\arctan(\mathrm{jy}) = \mathrm{j}\ \mathrm{artanh}\ \mathrm{y}$$

$$e^{jn\varphi} = \left(\cos\varphi + \sqrt{\cos^2\varphi - 1}\right)^n$$

$$\ln(-x) = \ln x \pm j\pi$$

$$\frac{1-\underline{z}}{1+\underline{z}} = \frac{1 - \mathrm{Re}^2\{\underline{z}\} - \mathrm{Im}^2\{\underline{z}\}}{\left(1 - \mathrm{Re}\{\underline{z}\}\right)^2 + \mathrm{Im}^2\{\underline{z}\}} - j\frac{2\,\mathrm{Im}\{\underline{z}\}}{\left(1 - \mathrm{Re}\{\underline{z}\}\right)^2 + \mathrm{Im}^2\{\underline{z}\}}$$

$$\ln\frac{a+jb}{a-jb} = j2\arctan\frac{b}{a}$$

$$\frac{a+jb}{a-jb} = \exp\left(j2\arctan\frac{b}{a}\right)$$

$$\ln(1+j) = \frac{1}{2}\ln 2 + j\frac{\pi}{4}$$

$$1+j = \sqrt{2}\,e^{j\pi/4}$$

$$\left|1+e^{j\varphi}\right| = 2\cos\frac{\varphi}{2}$$

$$\left|1-e^{j\varphi}\right| = 2\sin\frac{\varphi}{2}$$

$$\left|1+re^{j\varphi}\right| = \sqrt{1+2r\cos\varphi + r^2}$$

$$\measuredangle\left(1-e^{j\varphi}\right) = \frac{\varphi-\pi}{2}$$

$$\sqrt{a+jb} = \pm\left\{\sqrt{\frac{1}{2}\left(\sqrt{a^2+b^2}+a\right)} + j\sqrt{\frac{1}{2}\left(\sqrt{a^2+b^2}-a\right)}\right\} \qquad \text{für b} \geq 0$$

$$\sqrt{a+jb} = \pm\left\{\sqrt{\frac{1}{2}\left(\sqrt{a^2+b^2}+a\right)} - j\sqrt{\frac{1}{2}\left(\sqrt{a^2+b^2}-a\right)}\right\} \qquad \text{für b} < 0$$

VI. Elliptische Funktionen

K(k): vollständiges elliptisches Integral 1. Art, Modul k

$k' = \sqrt{1-k^2}$ $\qquad$ $K'(k) = K(k')$

$$\frac{K(k)}{K'(k)} \approx \frac{1}{\pi} \ln \frac{2+2\sqrt{k}}{1-\sqrt{k}} \qquad \text{für} \quad k \ge \frac{1}{\sqrt{2}}$$

Fehler < 3 10^{-6}

$$\frac{K(k)}{K'(k)} \approx \frac{\pi}{\ln \dfrac{2+2\sqrt{k'}}{1-\sqrt{k'}}} \qquad \text{für} \quad k \le \frac{1}{\sqrt{2}}$$

$$k \approx \left(\frac{\exp\left(\dfrac{\pi\, K(k)}{K'(k)}\right) - 2}{\exp\left(\dfrac{\pi\, K(k)}{K'(k)}\right) + 2} \right)^2 \qquad \text{für} \quad K(k) \ge K'(k) \text{ bzw. für} \quad k \ge \frac{1}{\sqrt{2}}$$

$$k' \approx \left(\frac{\exp\left(\dfrac{\pi\, K'(k)}{K(k)}\right) - 2}{\exp\left(\dfrac{\pi\, K'(k)}{K(k)}\right) + 2} \right)^2 \qquad \text{für} \quad K(k) \le K'(k) \text{ bzw. für} \quad k \le \frac{1}{\sqrt{2}}$$

$$k \approx \frac{4\sqrt{\exp\left(\dfrac{\pi K'(k)}{K(k)}\right)\left(\exp\left(\dfrac{2\pi K'(k)}{K(k)}\right) + 4\right)}}{\left(\exp\left(\dfrac{\pi K'(k)}{K(k)}\right) + 2\right)^2}$$

$$K(k) \approx \frac{2}{\left(1+\sqrt{k}\right)^2} \ln \frac{2+2\sqrt{k}}{1-\sqrt{k}} \qquad \text{für} \quad k \ge \frac{1}{\sqrt{2}}$$

$$K'(k) \approx \frac{2\,\pi}{\left(1+\sqrt{k}\right)^2} \qquad \text{für} \quad k \geq \frac{1}{\sqrt{2}}$$

$$K(k) \approx \frac{2\,\pi}{\left(1+\sqrt{k'}\right)^2} \qquad \text{für} \quad k \leq \frac{1}{\sqrt{2}}$$

$$K'(k) \approx \frac{2}{\left(1+\sqrt{k'}\right)^2} \ln \frac{2+2\sqrt{k'}}{1-\sqrt{k'}} \qquad \text{für} \quad k \leq \frac{1}{\sqrt{2}}$$

VII. Reihen

$$1+x+x^2+\ldots+x^{m-1}=\frac{1-x^m}{1-x}$$

$$1+2\sum_{n=1}^{(m-1)/2}\cos\left(n\pi\cos\varphi\right)=\frac{\sin\frac{m\pi}{2}\cos\varphi}{\sin\frac{\pi}{2}\cos\varphi}$$ m ungerade

$$1+2\sum_{n=1}^{(m-1)/2}\cos\left(n\varphi\right)=\frac{\sin\frac{m\varphi}{2}}{\sin\frac{\varphi}{2}}$$ m ungerade

$$2\sum_{n=1}^{m/2}\cos\left(\frac{2n-1}{2}\varphi\right)=\frac{\sin\frac{m\varphi}{2}}{\sin\frac{\varphi}{2}}$$ m gerade

VIII. Differentialgleichungen

$$\frac{d}{dx}\left(\frac{x+1}{x-1}\right)=-\frac{2}{(x-1)^2}$$

$$\frac{d}{dx}\left(\ln\frac{x+1}{x-1}\right)=-\frac{2}{x^2-1}$$

$$\frac{d}{dx}\left(\frac{x-1}{x+1}\right)=\frac{2}{(x+1)^2}$$

$$\frac{d}{dx}\left(\ln\frac{x-1}{x+1}\right)=\frac{2}{x^2-1}$$

$$\frac{d}{dx}\left(\frac{1-x}{1+x}\right)=-\frac{2}{(1+x)^2}$$

$$\frac{d}{dx}\left(\ln\frac{1-x}{1+x}\right)=-\frac{2}{1-x^2}$$

$$\frac{d}{dx}\left(\ln\frac{1+x}{1-x}\right)=\frac{2}{1-x^2}$$

$$\frac{d}{dx}\left(\ln\frac{1+jx}{1-jx}\right)=j\frac{2}{1+x^2}$$

$$\frac{d}{dx}\sqrt{\frac{x+a}{b-c}}=\frac{1}{2\sqrt{(x+a)(b+c)}}$$

mit a ≥ 0 oder a < 0